Projet ANR ALARICE

"Bornes de complexité générales pour les systèmes dynamiques finis"

♡ **Théorème [Rice, 1953].** Toute propriété nontriviale du comportement des programmes est indécidable.

Systèmes dynamiques finis donc décidables, donc complexité algorithmique.

Métathéorème [Objectif 1]. Toute propriété nontriviale de la dynamique des $\langle$mon_modèle_de_calcul$\rangle$ est $\mathcal{C}$-difficile.

Pour quelles définitions de *propriété nontriviale* et *classe* $\mathcal{C}$?

Plan

- ▷ https://alarice.lis-lab.fr/

  ↪ essayons de la maintenir à jour (publications, évènements) : git push ou mailto:kevin

- ▷ 3 Objectifs, 5 Tâches
- ▷ Budget et planning
- ▷ Metatéorèmes (EN)

3 Objectifs, 5 Tâches

Réseaux d'automates

$[n] = \{1, 2, \dots, n\}$

$f_i : \{0,1\}^n \to \{0,1\}$ for $i \in [n]$

Graphe d'interaction G_f sur les sommets $[n]$

Dynamique $\mathscr{G}_f$ sur les sommets $\{0,1\}^n$

- Objectif 1 : atteindre des métathéorèmes.
- Objectif 2 : épuiser les problèmes classiques.
- Objectif 3 : transposer à d'autres modèles de calcul.

- Tâche 0 : Coordination (rapports...). Adrien, Kévin
- Tâche 1 : Objectif 1 Metatheorems. Guillaume, Kévin
- Tâche 2 : Objectif 2 Classic. Adrien, Sylvain
- Tâche 3 : Objectif 3 Other Models. Enrico, Sara
- Tâche 4 : Diffusion (papiers, exposés, médiation). Florian, Kévin, Pierre

Budget et planning : 2025–2029 (5 ans)

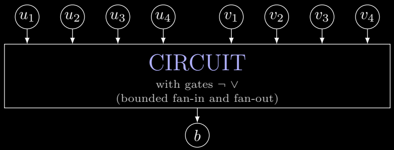
Publications sur HAL avec remerciement ANR-24-CE48-7504 (auto)

Metatheorems: results

Succinct graph representation of $\mathscr{G}_f$

$\hookrightarrow$ config/vertex label on $\log(|V|)$ bits

(nondeterministic)



[0] Gamard, Guillon, Perrot, Theyssier. "*Rice-Like Theorems for Automata Networks*". STACS'2021. hal-03430841

[1] Goubault-Larrecq, Perrot. "*Circuit Metaconstruction in Logspace for Rice-like Complexity Lower Bounds in ANs and SGRs*". CiE'2025. arxiv:2504.11348

[2] Goubault–Larrecq, Perrot. "*Rice-like Complexity Lower Bounds for Boolean and Uniform Automata Networks*". Preprint, 2025. arxiv:2409.08762

[3] Gamard, Goubault-Larrecq, Guillon, Ohlmann, Perrot, Theyssier. "*Hardness of Monadic Second-Order Formulae over Succinct Graphs*". LMCS (2026). 2302.04522

Definition. ψ is cw-nontrivial when there is $k \in \mathbb{N}$ such that ψ has ∞ many models and ∞ many counter-models of clique-width $\leq k$.

Theorem [3]. If ψ is a cw-nontrivial MSO sentence, then testing ψ on graphs represented succinctly is either NP-hard or coNP-hard.

Theorem [1]. The reduction can be computed in logspace.

Theorem [2]. The result holds for deterministic and q-uniform ANs.

If ψ is cw-nontrivial for sizes q^n then $\{q^n \mid n \in \mathbb{N}\} \cap \{ak + b \mid k \in \mathbb{N}\}$ contains a geom. seq.

[0] on deterministic FO with Hanf-locality, with more on limit sets.

Metatheorems: proof sketch $\frac{1}{3}$

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Proof sketch. Let ψ be cw-nontrivial of rank q.

1. Saturating graph
2. Pumping graph
3. SAT formula $S \mapsto$ circuit of $\mathscr{G}_f$

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$$\Omega_q = \bigsqcup_{q\text{-type } t} \bigsqcup^{p(q,t)} G_t \quad \text{verifies} \quad \forall G : G \sqcup \Omega_q \models \psi$$
$$\text{or } \forall G : G \sqcup \Omega_q \not\models \psi$$

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Compositionality lemma. For any $q \in \mathbb{N}$ and any operation, the output q-type depends only on the input q-types.

$\implies$ Label a clique-decomposition with q-types, and substitute.

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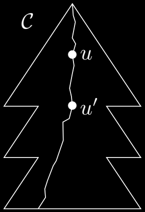

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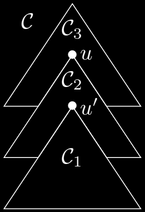

q-type at u = q-type at u'

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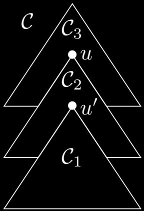

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Given S on s variables, construct a circuit for:

$$\mathcal{C}_1 \oplus \underbrace{\mathcal{C}_2/\Omega_q \oplus \mathcal{C}_2/\Omega_q \oplus \ldots \oplus \mathcal{C}_2/\Omega_q}_{2^s \text{ valuations of } S} \oplus \mathcal{C}_3 \models \psi \iff S \in \mathrm{SAT}$$

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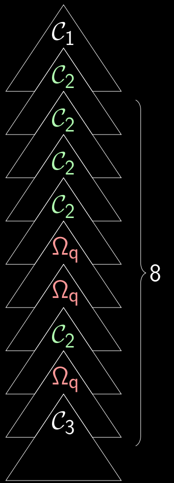
$S = (x_1 \vee \neg x_2) \wedge x_3$



Metatheorems: fresh results :-)

[4] Geniet, Goubault–Larrecq, Perrot. "*Complexity lower bounds for succinctly encoded binary structures of bounded clique-width, with restrictions*". Preprint, 2026+.

ψ-under-χ-dynamics
Input: a succinct R_2-graph representation (N, D).
Promise: $\mathscr{G}_{N,D} \models \chi$.
Question: does $\mathscr{G}_{N,D} \models \psi$?

χ can enforce $\leq\ \in R_2$ to be a partial or total order.

$\chi = \top$ is [3].

Theorem [4]. Let ψ and χ be MSO formulas over binary relations R_2.
If ψ is cw-nontrivial under restriction χ, then:

- ψ-under-χ-dynamics is NP-hard or coNP-hard if χ is union-stable;
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Definition. (ψ, χ) cw-size-independent means $|\mathrm{Spec}(\psi \wedge \chi) \cap \mathrm{Spec}(\neg\psi \wedge \chi)| = \infty$
$\implies$ we can have G^+ and G^- for arbitrary sizes.

Examples. Any ψ cw-nontrivial under $\chi =$ "clique or stable" is P-complete.
Union-unstable cw-size-dependent: $\chi =$ "cycle", $\psi =$ "bicolor" is N%2.

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Theorem [4]. Parameterization by twin-width or bounded degree fails.
There is a $\mathcal{P}_4$-nontrivial FO ψ such that, if ... then polytime SAT on robust set.

Metatheorems: open questions

Remark. If $\Omega_q \models \psi$ then NP-hard, if $\Omega_q \models \neg\psi$ then coNP-hard.
Remark. Deterministic have *tw* ≤ 2 so nontrivial/trivial dichotomy.

- From succinct graph representations to ANs ? $\implies$ arithmetics [3]
- Most comprehensive structural bounds to pump ? $\implies$ cw [4]
- Enrich the signature $\{=, \rightarrow\}$? $\implies$ under restriction χ [4]

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MSO ▷ Finer complexity lower bounds ?
Theorem [0]. Complete FO sentence ψ_i for each level Σ_i^P of PH.

[Balcázar, Lozano, Torán (1992)]		
	$\delta(G) \leq k$	Σ_2^P
	connectivity, planarity	PSPACE
	hamiltonian, k-colorability	NEXPTIME

MSO ▷ A cw-trivial MSO question on $\{=, \rightarrow\}$ in P ? (without restriction)

Circuit ▷ Pitfall of global nondeterminism: how to get local nondeterminism ?

Pump ▷ Transfer Rice-like complexity lower bounds to:
finite CAs, reaction systems, tile assembly, DNA folding, ...
via simulations acting as reductions. $\rightsquigarrow$ Simulation thesis

$\exists!$ ▷ What is the complexity of $\psi = \forall x : \exists! y : x \rightarrow y$? (determinism)

Extra: monadic second order graph logic and types

The variables are configurations (FO) or sets of configurations (MSO).

$$\psi \ := \ x{=}y \mid x{\to}y \mid x{\in}X \mid \psi{\vee}\psi \mid \neg\psi \mid \exists x : \psi \mid \exists X : \psi$$

Denote $\mathscr{G}_f \models \psi$ or $\mathscr{G}_f \not\models \psi$.

$\exists x : x \to x$ fixed-point

$\exists x^1, x^2, x^3 : (x^1 \to x^2) \wedge (x^2 \to x^3) \wedge (x^3 \to x^1)$ 3-cycle

$\forall x, x', y : [(x \to y) \wedge (x' \to y)] \Rightarrow (x = x')$ injectivity

$\forall x, y, y' : [(x \to y) \wedge (x \to y')] \Rightarrow (y = y')$ determinism

$\forall S : (\forall x : x \in S) \vee (\forall y : y \notin S) \vee (\exists x : \exists y : x \in S \wedge y \notin S \wedge x \to y)$ strongly connected

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strongly connected

The rank of ψ is its number of nested quantifiers q.

Proposition. Finite number of non-equivalent formulas of rank q.

The q-type of G is the set of formulas of rank q satisfied by G.

Proposition. Finite number of q-types for each $q \in \mathbb{N}$.

Extra: graph decompositions and compositionality

A k-graph $G = (V, E, C)$ has colors $C : V \to \{1, \ldots, k\}$.

$i(v)$	one vertex of color i (with or without self-loop)
$G \sqcup H$	disjoint union
$\rho_{i \to j}(G)$	recolor i as j
$\eta_{i,j}(\mathrm{G})$	insert arcs from color i to color j (with $i \neq j$)

A clique-decomposition of G is a tree labeled by these operations.

The clique-width of G is the minimum number of colors required.

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Theorem. $cw(G[V']) \leq cw(G)$ for any $V' \subseteq V$.
Theorem. $cw(G) \leq 3 \cdot 2^{tw(G)-1}$.
Theorem. Computing $cw(G)$ is NP-complete.
Theorem. Compute a $(8^k - 1)$ decomposition in time $\mathcal{O}(|V|^3)$.

Courcelle Olariu (2000), Corneil Rotics (2005), Fellows Rosamond Rotics Szeider (2009), Oum (2005)

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$\Longrightarrow$ Label a clique-decomposition with q-types, and substitute.

Extra: a counterpoint to Courcelle's theorem

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Theorem. For any MSO sentence ψ, testing ψ on G is computable in polynomial time on graphs of bounded clique-width.

Sketch. Compute a decomposition, then run a bottom-up tree automaton on q-types.

Model-checking with the adjacency matrix of G as input is "easy".

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[1]

Of course some questions are not hard, e.g. $\exists x, y : x \neq y$

Definition. ψ is trivial when it has finitely many models or finitely many countermodels.

$\implies$ time $\mathcal{O}(1)$.

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Courcelle Makowsky Rotics (2000)

Theorem. For any MSO sentence ψ, testing ψ on G is computable in polynomial time on graphs of bounded clique-width.

Sketch. Compute a decomposition, then run a bottom-up tree automaton on q-types.

Model-checking with the adjacency matrix of G as input is "easy".

Model-checking with a circuit for G as input is "hard".

Theorem. If ψ is a cw-nontrivial MSO sentence, then testing ψ on graphs represented succinctly is either NP-hard or coNP-hard.

[1]

Of course some questions are not hard, e.g. $\exists x, y : x \neq y$

Definition. ψ is cw-trivial when on graphs of bounded clique-width it has finitely many models or finitely many countermodels.

$\implies$ time $\mathcal{O}(1)$ on graphs of bounded clique-width.